Now, since the triangles PDK, FHM, and PEM are similar, and therefore proportional to the squares of their homologous sides, it follows that

$$\triangle PDK + \triangle FHM = \triangle PEM.$$

Adding the polygon $BFMPD$ to both sides of the above equivalence we get

$$\text{Area } BHK = \text{Area } BFED.$$

Remark. There will, in general, be two different solutions according as the side of the parallelogram drawn through P is taken parallel to BA or BC. If, however, the given area is such that $PE = PD$, the two solutions will be equal, and either solution will, in this case, give the *smallest triangle* that can be drawn with its side passing through P. If the given area is such that $PE < PD$, there will be no solution.

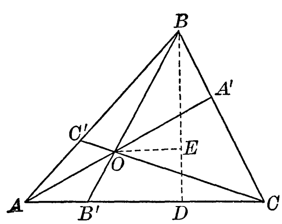
450. Proposed by W. L. WATSON, Moundsville, W. Va.

If three straight lines AA', BB', CC', drawn from the vertices of a triangle ABC to the opposite sides, pass through a common point O within the triangle, then

$$\frac{OA}{AA'} + \frac{OB}{BB'} + \frac{OC}{CC'} = 1.$$

SOLUTION BY MARCUS SKARSTEDT, Augustana College, Rock Island, Ill.

Draw BD perpendicular to AC, and OE perpendicular to DB. Then



$$\frac{OB'}{BB'} = \frac{ED}{BD} = \frac{\triangle AOC}{\triangle ABC}.$$

Similarly,

$$\frac{OA'}{AA'} = \frac{\triangle COB}{\triangle ABC} \quad \text{and} \quad \frac{OC'}{CC'} = \frac{\triangle ABO}{\triangle ABC}.$$

Adding, we get

$$\frac{OA'}{AA'} + \frac{OB'}{BB'} + \frac{OC'}{CC'} = \frac{\triangle COB + \triangle AOC + \triangle ABO}{\triangle ABC} = 1.$$

Solved similarly by A. M. HARDING, NATHAN ALTSHILLER, R. M. MATHEWS, A. H. HOLMES, T. DANTZIG, PAUL CAPRON, E. E. WHITFORD, HORACE OLSON, CLIFFORD N. MILLS, A. L. McCARTY, and GEORGE Y. SOSNOW.

451. Proposed by CLIFFORD N. MILLS, So. Dakota State College.

Determine the sides of an isosceles triangle of given area, having given that the sum of its sides is equal to the sum of its base and altitude.

SOLUTION BY ELIZABETH BROWN DAVIS, U. S. Naval Observatory.

Let BCD be the given isosceles triangle; A, its area; $2b$ its base; $2a$ its altitude; and c each of its equal sides. Then by the conditions of the problem

$$2c = 2a + 2b, \qquad \text{or} \qquad c = a + b.$$

Also,

$$(2a)^2 + b^2 = c^2 = (a + b)^2;$$

whence

$$3a^2 + 2ab = A,$$

and $a = \frac{1}{3}\sqrt{3A}$. Hence, $2a = \frac{2}{3}\sqrt{3A} =$ altitude. Since $2ab = 3a^2$, $b = \frac{3}{2}a = \frac{1}{2}\sqrt{3A}$; $2b = \sqrt{3A} =$ the base; and $c = a + b = \frac{5}{6}\sqrt{3A} =$ the length of the equal sides.

Also solved by C. E. GITHENS, ELBERT H. CLARKE, A. M. HARDING, A. H. HOLMES, WALTER C. EELLS, H. C. FEEMSTER, HORACE OLSON, GEORGE Y. SOSNOW, and NATHAN ALTSHILLER.

CALCULUS.

364. Proposed by EMMA GIBSON, Drury College.

Solve the differential equation

$$(xp - y)^2 = a(1 + p^2)(x^2 + y^2)^{3/2}, \text{ where } p = \frac{dy}{ax}.$$

I. SOLUTION BY GEO. W. HARTWELL, Hamline University.

Let $v = \dfrac{y}{x}$ and $u^2 = x^2 + y^2$.

The equation then takes such form that the variables can be separated and we have

$$\frac{dv}{1 + v^2} = \frac{\sqrt{a}\, du}{\sqrt{u - au^2}}.$$

Integrating,

$$\tan^{-1} v + c = \cos^{-1}(1 - 2au) = \operatorname{vers}^{-1} 2au.$$

$$\therefore\ \tan^{-1}\frac{y}{x} + c = \operatorname{vers}^{-1} 2a\sqrt{x^2 + y^2}.$$

II. SOLUTION BY C. C. STECK, New Hampshire College, Durham, N. H.

If we put $x = r\cos\theta$ and $y = r\sin\theta$ in the given equation we get

$$d\theta = \frac{adr}{\sqrt{ar - a^2r^2}}.$$

Integrating this we have

$$\theta + c = \text{arc vers } 2ar.$$

Whence,

$$\text{arc tan}\frac{y}{x} + c = \text{arc vers } 2a\sqrt{x^2 + y^2}.$$

Solved similarly by A. M. HARDING, C. N. SCHMALL, ELMER SCHUYLER, and LEROY COFFIN.